

國立臺灣師範大學資訊工程學系  
九十九學年度第一學期  
博士班資格考

考試科目：離散數學

總分一百分

請在答案卷作答，在題目卷上作答不予計分

1. (10 分) Please identify the error or errors in this argument that supposedly shows that if  $\exists x P(x) \wedge \exists x Q(x)$  is true then  $\exists x (P(x) \wedge Q(x))$  is true.  

(1) $\exists x P(x) \wedge \exists x Q(x)$	Premise
(2) $\exists x P(x)$	Simplification from (1)
(3) $P(c)$	Existential instantiation from (2)
(4) $\exists x Q(x)$	Simplification from (1)
(5) $Q(c)$	Existential instantiation from (4)
(6) $P(c) \wedge Q(c)$	Conjunction from (3) and (5)
(7) $\exists x (P(x) \wedge Q(x))$	Existential generation
2. (10 分) Please use a *non-constructive existence proof* to show that there exist irrational numbers  $x$  and  $y$  such that  $x^y$  is rational.
3. (10 分) Please determine whether each of these functions is one-to-one, onto, and bijection from  $\mathbf{Z}$  to  $\mathbf{Z}$ , where  $\mathbf{Z}$  denotes the set of all integers. Please explain your answer.
  - (a)  $f(n) = n-1$
  - (b)  $f(n) = n^2+1$
  - (c)  $f(n) = n^3$
  - (d)  $f(n) = \lceil n/2 \rceil$
4. (10 分) Please describe an efficient method to compute  $7^{160} \bmod 645$ .
5. (10 分) How many numbers must be selected from the set  $\{1, 3, 5, 7, 9, 11, 13, 15\}$  to guarantee that at least one pair of these numbers add up to 16. Please explain your answer.

6. (15 分) Suppose that  $E, F_1, F_2$ , and  $F_3$  are events from a sample space  $S$  and that  $F_1, F_2$ , and  $F_3$  are mutually disjoint and their union is  $S$ .

Find  $p(F_2 | E)$  if  $p(E | F_1)=2/7, p(E | F_2)=3/8, p(E | F_3)=1/2, p(F_1)=1/6, p(F_2)=1/2$ , and  $p(F_3)=1/3$ .

7. (15 分) Show that the Fibonacci numbers satisfy the recurrence relation  $f_n = 5f_{n-4} + 3f_{n-5}$  for  $n = 5, 6, 7, \dots$ , together with the initial conditions  $f_0 = 0, f_1 = 1, f_2 = 1, f_3 = 2$ , and  $f_4 = 3$ . Use this recurrence relation to show that  $f_{5n}$  is divisible by 5, for  $n = 1, 2, 3, \dots$ .

8. Short Answers:

- (a) (10 分) Let  $R$  be the relation on the set of real numbers such that  $xRy$  if and only if  $x$  and  $y$  are real numbers that differ by less than 1, that is  $|x - y| < 1$ . Show that  $R$  is not an equivalence relation.
- (b) (10 分) Draw the directed graph of the reflexive closure of the relations with the directed graph shown as follows.

