

# 國立臺灣師範大學 99 學年度碩士班招生考試試題

科目：數學基礎

適用系所：資訊工程學系

注意：1.本試題共 4 頁，請依序在答案卷上作答，並標明題號，不必抄題。2.答案必須寫在指定作答區內，否則不予計分。

1. (3 分) Translate the following statement into the logical expression using predicates, quantifiers, and logical connectives.

*Not everybody is your friend or someone is not perfect.*

2. (3 分) The successor of the set  $A$  is the set  $A \cup \{A\}$ . Find the successor of the set  $\{\emptyset, \{\emptyset\}\}$ .

3. (3 分) Find the value of  $\sum_{i=1}^4 \sum_{j=1}^3 ij$ .

4. (3 分) The value of the Euler  $\phi$ -function at the positive integer  $n$  is defined to be the number of positive integers less than or equal to  $n$  that are relatively prime to  $n$ . Please find  $\phi(10)$ .

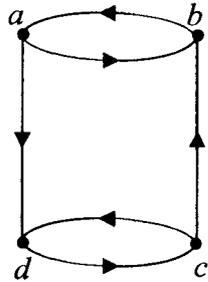
5. (3 分) Find the Boolean product of  $A$  and  $B$ , that is  $A \odot B$ , where

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}.$$

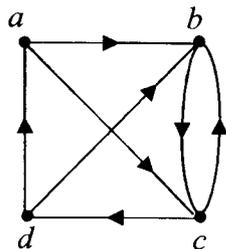
6. (3 分) Suppose that a popular style of running shoe is available for both men and women. The woman's shoe comes in sizes 6, 7, 8, and 9, and the man's shoe comes in sizes 8, 9, 10, 11, and 12. The man's shoe comes in white and black, while the woman's shoe comes in white, red, and black. Please determine the number of different shoes that a store has to stock to have at least one pair of this type of running shoe for all available sizes and colors for both men and women.
7. (5 分) Suppose that we have found that the word "Nike" occurs in 250 of 2000 messages known to be spam, and in 5 of 1000 messages known not to be spam. Estimate the probability that an incoming message containing the word "Nike" is spam, assuming that it is equally likely that an incoming message is spam or not spam. If our threshold for rejecting a message as spam is 0.9, will we reject this message?

國立臺灣師範大學 99 學年度碩士班招生考試試題

8. (5 分) Show that the sequence  $\{a_n = 7 \cdot 2^n - n + 2\}$  is a solution of the recurrence relation  $a_n = a_{n-1} + 2a_{n-2} + 2n - 9$ .
9. (5 分) Draw the directed graph of the reflexive closure of the relations with the directed graph shown as follows.



10. (5 分) Determine whether the following graph has an Euler circuit. Construct such a circuit when one exists. If no Euler circuit exists, determine whether the graph has an Euler path and construct such a path if one exists.



11. (6 分) Show that if  $n$  is an integer greater than 1, then  $n$  can be written as the product of primes.
12. (6 分) During a month with 30 days, a baseball team plays at least one game a day, but no more than 45 games in the month. Show that there must be a period of some number of consecutive days during which the team must play exactly 14 games.

國立臺灣師範大學 99 學年度碩士班招生考試試題

13.(4 分) Given that  $A = \begin{bmatrix} r & s & t \\ u & v & w \\ x & y & z \end{bmatrix}$  and  $\det(A) = 10$ , please evaluate the determinants

for the following matrices.

(a)  $\det(-2A)$

(b)  $\det([3A^{-1}]^T)$

(c)  $\det \begin{bmatrix} t & r & s \\ w & u & v \\ z & x & y \end{bmatrix}$

14. Consider the following three matrices:

$$A = \begin{bmatrix} 1 & 4 & 6 \\ 0 & 0 & 1 \\ 2 & 10 & 9 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 4 & 6 \\ 0 & 0 & 1 \\ 0 & 2 & -3 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 4 & 6 \\ 0 & 2 & -3 \\ 0 & 0 & 1 \end{bmatrix}$$

(a) (3 分) Please find elementary matrices  $E_1, E_2, E_3$ , and  $E_4$  satisfying the given equations:  $E_1A = B, E_2B = A, E_3B = C, E_4C = B$ .

(b) (3 分) Please find the inverse of  $A$  by using Gauss-Jordan elimination.

15. Consider the bases  $B = \{(1, 0), (1, 1)\}$  and  $B' = \{(4, 5), (2, 3)\}$  for  $R^2$ .

(a) (3 分) Find the change-of-coordinate matrix  $A$  from  $B$  to  $B'$ .

(b) (3 分) Please find  $[(11, 14)]_B$  and get  $[(11, 14)]_{B'}$  according to the answer of (a).

16. A linear transformation  $T: R^2 \rightarrow R^2$  is defined by  $T(x, y) = (x-2y, 2x+3y)$ .

(a) (3 分) Please find a basis for the range of  $T$ .

(b) (3 分) Please find a basis for the kernel of  $T$ .

(c) (3 分) Please find the rank and nullity of  $T$ .

國立臺灣師範大學 99 學年度碩士班招生考試試題

17. (5 分) Please determine whether the following set of vectors in  $R^4$  is linear independent.

$$(1, a, 0, 0), (0, 2, a, 0), (0, 0, 3, a), (0, 0, 0, 4)$$

Give reasons for your answer.

18. (5 分) Please determine whether the following vectors span  $R^3$ .

$$(1, 5, 1), (5, 1, -1), (3, 3, 0), (10, 14, 1)$$

Give reasons for your answer.

19. (5 分) Let  $A$  be a  $5 \times 5$  matrix with characteristic equation  $\lambda(\lambda+5)^3(\lambda-8) = 0$ .

Besides, it is known that  $A\mathbf{u} = -5\mathbf{u}$ , where  $\mathbf{u}$  is in  $R^5$ . What are the possible dimensions of the solution space of  $\mathbf{u}$ ? Give reasons for your answer.

20. (5 分) Assume that  $b \neq 0$ , please find a matrix  $P$  that orthogonally diagonalizes  $B$ , and determine  $P^{-1}BP$ .

$$B = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$$

21. (5 分) Let  $w$  be the line in  $R^3$  with parameter equations  $x=2t, y=-t, z=4t$ .

Please find the matrix  $M$  such that the orthogonal projection of a point  $P(x_0, y_0, z_0)$

$$\text{onto } w \text{ is } \text{Proj}_w P = M \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix}.$$