

國立臺灣師範大學九十七學年度碩士班考試入學招生試題

數學基礎 科試題 (資訊工程學系用, 本試題共 4 頁)

注意: 1. 依次序作答, 只要標明題號, 不必抄題。
2. 答案必須寫在答案卷上, 否則不予計分。

一、是非題：請用“○”代表“是”，“×”代表“非”。

(15 題，每題 2 分，共 30 分)

For all problems, you may assume all matrices and vectors are of appropriate sizes.

1. If a system $A\mathbf{x}=\mathbf{b}$ has more than one solution, then so does the system $A\mathbf{x}=\mathbf{0}$.
2. If B is produced by interchanging two rows of A , then $\det B = \det A$.
3. If B is produced by multiplying row 3 of A by 5, then $\det B = 5 \det A$.
4. If $AC = 0$, then either $A = 0$ or $C = 0$.
5. If V is a nonzero finite-dimensional vector spaces, and if every set of p elements in V
6. If A and B are square and invertible, then AB is invertible, and $(AB)^{-1}=A^{-1}B^{-1}$.
fails to span V , then $\dim V > p$.
7. The non-pivot columns of a matrix are always linearly dependent.
8. A least-squares solution of $A\mathbf{x}=\mathbf{b}$ is a vector $\hat{\mathbf{x}}$ such that $\|\mathbf{b} - A\mathbf{x}\| \leq \|\mathbf{b} - A\hat{\mathbf{x}}\|$ for all \mathbf{x} in \mathbb{R}^n .
9. Row operations on a matrix A can change the linear dependence relations among the rows of A .
10. If A is invertible, then $\det A^{-1} = \det A$.
11. If $\{\mathbf{u}, \mathbf{v}\}$ is an orthonormal set in V , then $\|\mathbf{u} - \mathbf{v}\| = \sqrt{2}$.
12. Two eigenvectors corresponding to the same eigenvalue are always linearly dependent.
13. If $\|\mathbf{u} + \mathbf{v}\|^2 = \|\mathbf{u}\|^2 + \|\mathbf{v}\|^2$, then \mathbf{u} and \mathbf{v} are orthogonal.

14. An $n \times n$ matrix with n linearly independent eigenvectors is invertible.
15. If $AP=PD$, with D diagonal, then the nonzero columns of P must be eigenvectors of A .

二、簡答題：(14 題，共 70 分)

1. (5 分) Which of the following are bases for \mathbf{R}^3 ? _____
- (A) $(1, 2, 0)$ and $(0, 1, -1)$
- (B) $(1, 1, -1), (2, 3, 4), (4, 1, -1), (0, 1, -1)$
- (C) $(1, 2, 2), (-1, 2, 1), (0, 8, 0)$
- (D) $(1, 2, 2), (-1, 2, 1), (0, 8, 6)$
2. (5 分) What is the area of a triangle whose corners are $(2, 1), (3, 4)$, and $(0, 5)$?
3. (5 分) If a corner $(-1, 0)$ is added to the triangle above to make it a lopsided region (four sides). What is the area of this four sided polygon?
4. (5 分) Let λ be an eigenvalue of an invertible matrix A . Show that λ^{-1} is an eigenvalue of A^{-1} .
5. (3 分) The Fibonacci numbers are the numbers in sequence $0, 1, 1, 2, 3, 5, 8, 13, \dots$, where each term is the sum of the previous two. Let F_n denote the n th Fibonacci number. Then, we obtain the relationship $F_n = F_{n-1} + F_{n-2}$. Find the smallest Fibonacci number (> 1), which is a perfect square.
6. (3 分) Let $A = \{1, 3, 5\}$ and $B = \{1, 3, 5, 7, 9\}$. Find all sets C satisfying $A \cup C = B \cap C$.
7. (4 分) Find the number of terms in the expansion of $(w + x + y + z)^{12}$.
8. (4 分) Suppose a connected planar graph G has five vertices of degree 2, two vertices of order 3, and three vertices of order 4. Find the number of faces G has.

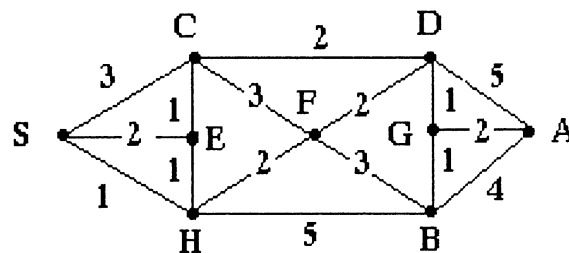
9. (5 分) If two fair dice are rolled and X_i is the face appearing on the i th dice, $i = 1, 2$, find the probability $P(X_1^2 + X_2^2 < 10)$.

10. (5 分) Solve the following homogeneous difference equation

$$y_n = 3y_{n-1} - 3y_{n-2} + y_{n-3} \quad \text{subject to the boundary values}$$

$$y_0 = 0, \quad y_3 = 3, \quad y_5 = 10.$$

11. (6 分) Referring to the following weighted graph, find a shortest path and the associated distance from the starting node S to (a) node A and (b) node B , respectively.



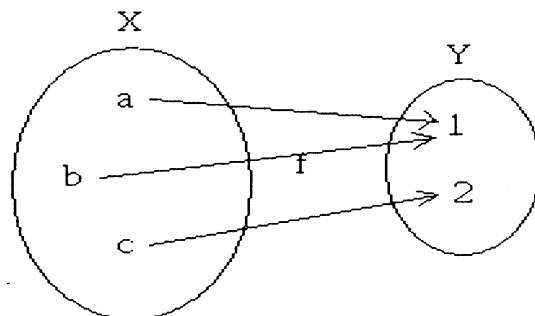
12. (6 分) Simplify the following Boolean functions:

(a) $f(x, y, z) = xy + y\bar{z} + x\bar{y} + \bar{x}yz$, and (b) $f(x, y, z) = xy + xz + \bar{y}z$.

13. (6 分) Give a regular expression for the language (over the alphabet $\{x, y\}$) of all strings, which (a) either begin with xx or end with yy ; (b) do not contain the substring xy .

14. (8 分) Let $X = \{a, b, c\}$ and $Y = \{1, 2\}$ be two sets. Define a

function $f: X \rightarrow Y$ as shown below.



Let $P(X)$ and $P(Y)$ denote the power sets of X and Y , respectively. We apply the function f to the power sets $P(X)$ and $P(Y)$ as $f: P(X) \rightarrow P(Y)$ and define

$$f(A) = \{y \mid y = f(x), x \in A\}, \text{ where } A \in P(X), \text{ and}$$

$$f^{-1}(B) = \{x \mid f(x) \in B\}, \text{ where } B \in P(Y).$$

What are (a) $f(\{a, b\})$, and (b) $f^{-1}(\{1, 2\})$?