

國立臺灣師範大學九十四學年度碩士班考試入學招生試題

數學基礎 科試題 (資訊工程研究所用, 本試題共 4 頁)

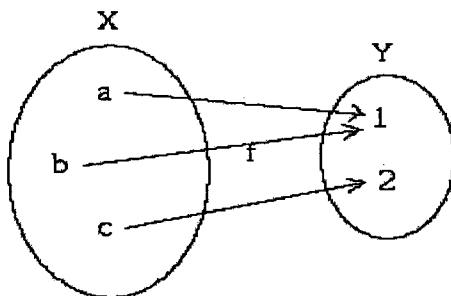
注意: 1. 依次序作答, 只要標明題號, 不必抄題。
2. 答案必須寫在答案卷上, 否則不予計分。

1. How many functions are there from $\{+, -, \times, \div, *\}$ to $\{a, b, c, d, e\}$? (3 分)
2. How many one-to-one functions are there from $\{+, -, \times, \div, *\}$ to $\{1, 2, 3, 4, 5\}$? (4 分)
3. How many permutations of the letters "CCCCDEEEE" read the same backwards as forwards? (5 分)
4. Suppose a connected planar graph G has five vertices of degree 2, two vertices of order 3, and three vertices of order 4. Find the number of faces G has. (5 分)
5. Is the graph whose adjacent matrix is given below planar? Justify your answer. (6 分)

0	1	1	1	0	1
1	0	0	1	1	1
1	0	0	0	1	0
1	1	0	0	1	1
0	1	1	1	0	1
1	1	0	1	1	0

6. Find all the roots of the characteristic equation of the difference equation $x_n - 4x_{n-1} + 3x_{n-2} = 0$. (10 分)

7. Let $X = \{a, b, c\}$ and $Y = \{1, 2\}$ be two sets. Define a function $f: X \rightarrow Y$ as shown below.



Let $P(X)$ and $P(Y)$ denote the power sets of X and Y , respectively. We apply the function f to the power sets $P(X)$ and $P(Y)$ as $f: P(X) \rightarrow P(Y)$ and define

$$f(A) = \{y \mid y = f(x), x \in A\}, \text{ where } A \in P(X), \text{ and}$$

$$f^{-1}(B) = \{x \mid f(x) \in B\}, \text{ where } B \in P(Y).$$

What are (a) $f(\{a, b\})$ (5 分), and (b) $f^{-1}(\{1, 2\})$? (5 分)

8. Let $T: P_2 \rightarrow P_2$ be defined by

$$T(a_0 + a_1x + a_2x^2) = 2(a_1 - a_2) + (2a_0 + 3a_2)x + 3a_2x^2$$

(a) Let $B = \{1, x, x^2\}$ be a basis for P_2 . Give the matrix of T with respect to

B . (4 分)

(b) Find the eigenvectors and the associated eigenvalues for T . (6 分)

(c) Let C denote the basis of P_2 that consists of the eigenvectors for T . Give

the matrix of T with respect to C . (5 分)

9. Let W be an m -dimensional subspace of R^n with basis $\{u_1, u_2, \dots, u_m\}$. Let

matrix $P = [u_1, u_2, \dots, u_m]$, from which we construct matrix

$$A = P(P^T P)^{-1} P^T.$$

(a) Calculate $A^2 - A$. (4 分)

(b) If A is an invertible matrix, what is A ? (5 分)

(c) If λ is an eigenvalue of A , what is λ ? (5 分)

10. Suppose A is an n by n matrix and x is an n by 1 vector. Give the condition that the length of Ax equals the length of $A^T x$. (4 分)

11. (a) Find the straight line $y = A + Bx$ that best fits the measurements, (x, y) , of $(0, 0)$, $(1, 1)$, $(3, 2)$ and $(4, 5)$. (4 分)

(b) Find the curve $y = C + D2^x$ that gives the best least squares fit to the measurements, (x, y) , of $(0, 6)$, $(1, 4)$ and $(2, 0)$. (5 分)

12. Referring to the figure given below, suppose that the planes P and P' are perpendicular to each other. There are two parallel lines L_1 and L_2 lying on the plane P . The point O has the distances of h and f from the planes P and P' , respectively. Let L'_1 and L'_2 be the projections of L_1 and L_2 onto the plane P' with respect to the point O . Let I specify the intersection point of lines L'_1 and L'_2 on the plane P' . Compute the distance between point I and plane P . (15 分)

