

國立臺灣師範大學 99 學年度學士班二年級轉學生招生考試試題

科目：微積分

適用學系(組)：資訊工程學系

注意：1. 本試題共 1 頁，請依序作答，並標明題號，不必抄題。

2. 答案必須寫在答案卷上之指定作答區內，否則依規定予以扣分。

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1. (i) (6 points) Find  $dy^2/dx^2$  as a function of  $t$  if  $x = t - t^2$ ,  $y = t - t^3$ ; (ii) (6 points) Find an equation for the tangent line to the curve at the point defined by the given value  $t = 2$ .
2. (10 points) Jane is 2 km offshore in a boat and wishes to reach a coastal village 6 km down a straight shoreline from the point nearest the boat. Since she can row 2 km/hour and can walk 5 km/hour. Where should she land her boat to reach the village in the least amount of time?
3. (i) (8 points) Find the area of the region in the first quadrant that is bounded above by  $y = 2\sqrt{x}$  and below by the  $x$ -axis and the line  $y = 2x - 4$ . (ii) (8 points) The region in the first quadrant that is enclosed by the  $x$ -axis and the curve  $y = 3x\sqrt{1-x}$  is revolved about the  $y$ -axis to generate a solid. Find the volume of the solid.
4. (i) (8 points) Find a curve through the origin in the  $xy$ -plane whose length from  $x = 0$  to  $x = 1$  is  $L = \int_0^1 \sqrt{1 + \frac{1}{4}e^x} dx$ . (ii) (8 points) The function  $f(x) = e^x + 2x$ , being differentiable and one-to-one, has a differentiable inverse  $f^{-1}(x)$ . Find the value of  $\frac{df^{-1}}{dx}$  at point  $f(\ln 2)$ .
5. (i) (6 points) Find  $\int (\ln x + e^x \cos x) dx$ . (ii) (4 points) Find  $\lim_{x \rightarrow 0^+} \frac{\sqrt{2x}}{\sqrt{\sin(3x)}}$ . (iii) (4 points) Find  $\lim_{x \rightarrow \infty} (\ln x)^{3/x}$ . (iv) (6 points) Solve the initial value problem:  
 $(t^2 + 2t) \frac{dx}{dt} = 2x + 2$  ( $t, x > 0$ ),  $x(1) = 1$ . (v) (6 points) Find a value for the constant  $b$  that will make the radius of convergence of the power series  $\sum_{n=2}^{\infty} \frac{b^n x^n}{\ln n}$  equal to 5.
6. (10 points) Find the Taylor series generated by  $f(x) = 2^x$  at  $x = 1$ .
7. (10 points) Find (i) the vector projection of  $\mathbf{u} = 4\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$  onto  $\mathbf{v} = \mathbf{i} - 2\mathbf{j} - 2\mathbf{k}$  and (ii) the scalar component of  $\mathbf{u}$  in the direction of  $\mathbf{v}$ . (Note that  $\mathbf{i}$ ,  $\mathbf{j}$ , and  $\mathbf{k}$  denote the standard unit vectors.)